1.5 Determine if the following CT signals are periodic. If yes, calculate the fundamental period $T_0$:

a) $x_2(t) = |\sin(-\frac{5\pi t}{8} + \frac{\pi}{2})|$

b) $x_4(t) = \exp(j(5t + \frac{\pi}{4}))$

c) $x_6(t) = 2\cos\left(\frac{4\pi t}{5}\right)\sin^2\left(\frac{16t}{3}\right)$

a) $x_2(t) = |\sin(-\frac{5\pi t}{8} + \frac{\pi}{2})|$

$\cos\left(\frac{5\pi t}{8} + \frac{5\pi T}{8}\right) = x_2(t)$ if $5\pi T/8$ or if $T=8/5$

b) $x_4(t) = \exp(j(5t + \frac{\pi}{4}))$

All CT complex exponentials are periodic

$x_4(t)$ is periodic with fundamental period $T=2\pi/5$

c) $x_6(t) = 2\cos\left(\frac{4\pi t}{5}\right)\sin^2\left(\frac{16t}{3}\right)$

$x_6(t) = 2\cos\left(\frac{4\pi t}{5}\right)\sin^2\left(\frac{16t}{3}\right) = 2\cos\left(\frac{4\pi t}{5}\right)\left(\frac{1}{2}\right)\left(1 - \cos\left(\frac{32t}{3}\right)\right)$

$= \cos\left(\frac{4\pi t}{5}\right) - \cos\left(\frac{4\pi t}{5}\right)\cos\left(\frac{32t}{3}\right)$

$= \cos\left(\frac{4\pi t}{5}\right) - \left(\frac{1}{2}\right)\left[\cos\left(\frac{4\pi}{5} - \frac{32t}{3}\right)\right] + \cos\left(\frac{4\pi}{5} + \frac{32t}{3}\right)$

$= \cos\left(\frac{4\pi t}{5}\right) - \left(\frac{1}{2}\right)\left[\cos\left(\frac{12\pi - 160}{15}\right) - \frac{1}{2}\right] + \left(\frac{1}{2}\right)\cos\left(\frac{12\pi + 160}{15}\right)$

Periods $T_1=5/2$, $T_2=(30\pi)/(12\pi-160)$, $T_3=-(30\pi)/(12\pi+160)$

$x_6(t)$ is periodic if all combinations $T_1/T_2$, $T_1/T_3$, $T_2/T_3$ are rational numbers.

Since $T_1/T_2$ is not a rational number, then $x_6(t)$ is not periodic.

1.9 Show that the average power of the CT period signal $x(t) = A\sin(\omega_0 t + \theta)$ with real-valued coefficient $A$, is given by $A^2/2$.

The signal has a period $T_0=2\pi/\omega_0$.

$$P_x = \left(\frac{1}{T_0}\right)\int_0^{T_0} \sin^2(\omega_0 t + \theta) dt = \left(\frac{A^2}{2T_0}\right)\int_0^{T_0} \left[1 - \cos(2\omega_0 t + 2\theta)\right] dt$$
\[ = \left( \frac{A^2}{2} \right) \left[ T_0 \right]_0^T \left[ \frac{\sin (2\omega_0 t + 2\theta)}{4T_0\omega_0} \right]_0^T \text{ since } T_0=2\pi/\omega_0 \text{ then we will have:} \]
\[ = \left( \frac{A^2}{2} \right) - \left( \frac{A^2}{4T_0\omega_0} \right) [\sin(2\theta + 4\pi) - \sin(2\theta)] \]
\[ = \frac{A^2}{2} - \frac{A^2}{4T_0\omega_0} [\sin(2\theta) - \sin(2\theta)] = \frac{A^2}{2} \]

**1.16** Consider the following signal:
\[ x(t) = 3\sin \left( \frac{2\pi(t - T)}{5} \right) \]

Determine the values of T for which the resulting signal is (a) an even function, and (b) an odd function of the independent variable t.

a) For \( x(t) \) to be an even function then \( x(t) = x(-t) \) so \( T = T_e \),
\[ 3 \sin \left( \frac{2\pi t - 2\pi T_e}{5} \right) = 3 \sin \left( \frac{-2\pi t}{5} - \frac{2\pi T_e}{5} \right) = -3 \sin \left( \frac{2\pi t}{5} + \frac{2\pi T_e}{5} \right) \]

\[ \frac{x(t)}{x(-t)} \]

\[ 3 \sin \left( \frac{2\pi t - 2\pi T_e}{5} \right) = 3 \sin \left( \frac{2\pi t}{5} + \frac{2\pi T_e}{5} + (2m + 1)\pi \right) \]

By matching terms we will have:
\[ -\frac{2\pi T_e}{5} = \frac{2\pi T_e}{5} + (2m + 1)\pi \text{ so } T_e = 5(2m+1)/4 \]

b) For \( x(t) \) to be an odd function then \( x(t) = -x(-t) \) so \( T = T_o \)
\[ 3 \sin \left( \frac{2\pi t - 2\pi T_o}{5} \right) = -3 \sin \left( \frac{-2\pi t}{5} - \frac{2\pi T_o}{5} \right) = 3 \sin \left( \frac{2\pi t}{5} + \frac{2\pi T_o}{5} \right) \]

\[ \frac{x(t)}{-x(-t)} \]

\[ 3 \sin \left( \frac{2\pi t - 2\pi T_o}{5} \right) = 3 \sin \left( \frac{2\pi t}{5} + \frac{2\pi T_o}{5} - (2m)\pi \right) \]

By matching terms we will have:
\[ -\frac{2\pi T_o}{5} = \frac{2\pi T_o}{5} - (2m)\pi \text{ so } T_o = 5m/2 \]

**1.18** Sketch the following CT signals:

a) \( x_1(t) = u(t) + 2u(t - 3) - 2u(t - 6) - u(t - 9) \);

b) \( x_3(t) = \text{rect} \left( \frac{t}{6} \right) + \text{rect} \left( \frac{t}{4} \right) + \text{rect} \left( \frac{t}{2} \right) \);

c) \( x_5(t) = (\exp(-t) - \exp(-3t))u(t) \)
1.20 Sketch the following DT signals:

a) \( x_1[k] = u[k] + u[k - 3] - u[k - 5] - u[k - 7] \)
b) \( x_3[k] = (3^k - 2^k)u[k] \)
c) \( x_5[k] = ku[k] \)
1.25 Consider the function \( f(t) \) shown in Fig. P1.25.

i) Sketch the function \( g(t) = f(-3t+9) \)

ii) Calculate the energy and power of the signal \( f(t) \). Is it a power signal or an energy signal?
    The \( f(t) \) is a finite signal so it is an energy signal. The average power of \( f(t) = 0 \).
    The total energy is:

\[
E = \int_{-\infty}^{\infty} f^2(t) dt = \int_{-3}^{0} (-t - 3)^2 dt + \int_{0}^{3} \left( \frac{5}{3} t - 3 \right)^2 dt
\]

\[
E = \int_{-3}^{0} (t^2 + 6t + 9) dt + \int_{0}^{3} \left( \frac{25t^2}{9} - 10t + 9 \right) dt = 16
\]

(iii) Repeat (ii) for \( g(t) \).

The function \( g(t) \) can be represented as \( g(t) = \begin{cases} 
-5t + 12 & 2 \leq t \leq 3 \\
3t - 12 & 3 \leq t \leq 4
\end{cases} \)

Since \( g(t) \) is a finite signal, it is an energy signal. The average power of \( g(t) = 0 \).
The total energy is:
\[
E = \int_{-\infty}^{\infty} g^2(t) \, dt = \int_{2}^{3} (-5t + 12)^2 \, dt + \int_{3}^{4} (3t - 12)^2 \, dt
\]
\[
E = \int_{-3}^{0} (25t^2 - 120t + 144) \, dt + \int_{0}^{3} (9t^2 - 72t + 144) \, dt = 16/3
\]

1.31 (MATLAB exercise) Write a set of MATLAB functions that compute and plot the following CT signals. In each case, use a sampling interval of 0.001s.

(i) \( x(t) = \exp(-2t) \sin(10\pi t) \) for \(|t| \leq 1\).

(ii) A periodic signal \( x(t) \) with fundamental period \( T=5 \). The value over one period is given by
\[
x(t) = 5t \quad 0 \leq t < 5
\]
Use the sawtooth function available in MATLAB to plot five periods of \( x(t) \) over the range -10 \( \leq t < 15 \).  

(iii) The unit step function \( u(t) \) over \([-10, 10]\) using the sign function available in MATLAB.

% part (i)
t = -1:0.001:1; x = exp(-2*t).*sin(10*pi*t);
subplot(221); plot(t,x); xlabel('t');
title('(i) \exp(-2t) \sin(10\pi t)');
grid;

% part (ii)
t = -10:0.001:15; x = sawtooth(2*pi*t/5);
subplot(222); plot(t,x); xlabel('t');
title('(ii) Sawtooth wave with a period of 5s');
grid;

% part (iii)
t = -10:0.001:10; x = 0.5*(1 + sign(t));
subplot(223); plot(t,x); xlabel('t');
title('(iii) u(t)');
grid; axis([-10 10 -0.1 1.1]);