1. Find the truth table for the following functions:

(a) \( F = y'z' + y'z + xz' \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
<th>( y'z' )</th>
<th>( y'z )</th>
<th>( xz' )</th>
<th>( y'z' + y'z + xz' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>1</td>
<td>0 0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 0 1</td>
<td>0</td>
<td>0 1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 1 0</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 1 1</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 0 0</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 0 1</td>
<td>0</td>
<td>0 0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1 0</td>
<td>0</td>
<td>0 0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1 1</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) \( F = xy + x'z' \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
<th>( xy )</th>
<th>( x'z' )</th>
<th>( xy + x'z' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>0</td>
<td>1</td>
<td>0 0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0 0 1</td>
<td>0</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0 1 0</td>
<td>0</td>
<td>1</td>
<td>1 0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0 1 1</td>
<td>0</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1 0 0</td>
<td>1</td>
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<td>0 0</td>
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<td></td>
</tr>
<tr>
<td>1 0 1</td>
<td>1</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1 1 0</td>
<td>1</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1 1 1</td>
<td>1</td>
<td>0</td>
<td>0 0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
2. Implement the Boolean function \( F = yz + y'z' + z'z \)

This can be reduced to \( F = yz + y'z' \).

(a) with AND, OR and inverter gates,

(b) with NAND and inverter gates,
(c) with NOR and inverter gates.

Convert from sum of products to product of sums: \((y + z')(y' + z) = ((y + z')' + (y' + z)')'\)

3. Obtain the truth table of the following functions, and express each function as a sum-of-minterms and a product-of-maxterms:

(a) \((x + yz)(z + xz)\)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>(x + yz)</th>
<th>(z + xz)</th>
<th>(xyz)(z + xz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The minterms are the ones with 1’s:
\(\Sigma(3, 5, 7)\)
The maxterms are the ones with the 0’s:
\(\Pi(0, 1, 2, 4, 6)\)
(b) \((xy' + yz + x'y)(x + y)\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(z)</th>
<th>((xy' + yz + x'y))</th>
<th>(x + y)</th>
<th>((xy' + yz + x'y)(x + y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The minterms are where there are 1’s:
\(F = \Sigma(2, 3, 4, 5, 7)\)

The maxterms are where there are 0’s:
\(F = \Pi(0, 1, 6)\)

4. Express the following function as a sum of minterms and as a product of maxterms:

\[ F(A, B, C, D) = AC + BD' + BC' + BD \]

\[ F(A, B, C, D) = AC + BD' + BC' + BD \]
\[ = AC + B(D' + D) + BC' \]
\[ = AC + B + BC' \]
\[ = AC + B \]

Truth table:

<table>
<thead>
<tr>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(AC + B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0</td>
<td>0 0 0 1</td>
<td>0 0 1 0</td>
<td>0 0 1 1</td>
<td>0 1 0 0</td>
</tr>
</tbody>
</table>

\[ F(A, B, C, D) = \Sigma(4, 5, 6, 7, 10, 11, 12, 13, 14, 15) \]
\[ F(A, B, C, D) = \Pi(0, 1, 2, 3, 8, 9) \]
5. Convert each of the following to the other canonical form:

(a) \( F(x, y, z) = \Sigma(1, 3, 6) \)
\[ F(x, y, z) = \Pi(0, 2, 4, 5, 7) \]
(b) \( F(A, B, C, D) = \Pi(0, 2, 4, 7, 9, 13) \)
\[ F(A, B, C, D) = \Sigma(1, 3, 5, 6, 8, 10, 11, 12, 14, 15) \]

6. Convert each of the following expressions into sum of products and products of sums:

(a) \( (BC + D)(C + AD) \)
\[ F = (BC + D)(C + AD) \]
\[ = BC + AD + D + CD + ADD' \]
\[ = BC + CD \] (Sum of Products form)
\[ = (B + D)C \] (Product of Sums form)
(b) \( y' + y(y + z')(x' + z) \)
Note: Below we use
\[ y' + yz = y'(z + z') + yz = y'(z + z') + yz = y' + (y' + y)z = y' + z \]
in two places.
\[ F = y' + y(y + z')(x' + z) \]
\[ = y' + x'y + yz + x'yz' + yzz' \]
\[ = y' + x'y(1 + z') + yz \]
\[ = y' + x'y + yz \]
\[ = y' + xy' + y' + yz \]
\[ = y' + x' + y' + z \]
\[ = y' + x' + z \] (SOP and POS)

7. Simplify the following Boolean functions using three-variable maps:

(a) \( F(x, y, z) = \Sigma(0, 3, 4, 5, 6, 7) \)
\[
\begin{array}{cccc}
  & y & z \\
  x & 00 & 01 & 11 & 10 \\
 0 & 0 & 1 & 3 & 2 \\
 1 & 4 & 5 & 1 & 6 \\
\end{array}
\]
\[ F = x + yz + y'z' \]
(b) \( F(x, y, z) = \Pi(3, 5, 7) \)
\[
\begin{array}{cccc}
  & y & z \\
  x & 00 & 01 & 11 & 10 \\
 0 & 1 & 1 & 3 & 0 \\
 1 & 4 & 5 & 0 & 1 \\
\end{array}
\]
\[ F = z' + x'y' \]
(c) \( F(x, y, z) = \Sigma(0, 2, 5, 7) \)

\[
\begin{array}{c|cccc}
\text{x} & 00 & 01 & 11 & 10 \\
\hline
0 & 0 & 1 & 0 & \boxed{3} \\
1 & 4 & 0 & \boxed{5} & 1 \end{array}
\]

\[ F = x'z' + xz \]

8. Simplify the following Boolean expressions using three-variable maps:

(a) \( F(x, y, z) = x'y'z' + yz + x'y'z \)

\[
\begin{array}{c|cccc}
\text{x} & 00 & 01 & 11 & 10 \\
\hline
0 & 0 & \boxed{1} & 1 & \boxed{3} \\
1 & 4 & 0 & 5 & \boxed{7} \end{array}
\]

\[ F = x'y' + yz \]

(b) \( F(x, y, z) = xy + y'z' + x'y'z \)

\[
\begin{array}{c|cccc}
\text{x} & 00 & 01 & 11 & 10 \\
\hline
0 & 0 & \boxed{1} & 1 & \boxed{3} \\
1 & 4 & 1 & 5 & 0 \end{array}
\]

Two ways:
\[ F = xy + x'y' + y'z' \]

or
\[ F = xy + x'y' + xz' \]
(c) $F(x, y, z) = x'y + y'z + x'z'$

<table>
<thead>
<tr>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00</td>
</tr>
<tr>
<td>1</td>
<td>01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>00</th>
<th>01</th>
<th>11</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

$F = x' + y'z$

(d) $F(x, y, z) = xyz + xy'z' + x'yz'$

<table>
<thead>
<tr>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00</td>
</tr>
<tr>
<td>1</td>
<td>01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>00</th>
<th>01</th>
<th>11</th>
<th>10</th>
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<tbody>
<tr>
<td>0</td>
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<td>3</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

Cannot reduce

$F(x, y, z) = xyz + xy'z' + x'yz'$
9. Simplify the following Boolean functions, using Karnaugh maps:

(a) \( F(A, B, C, D) = \Sigma(1, 5, 6, 7, 11, 12, 13, 15) \)

\[
\begin{array}{c|cccc}
& CD \\
A B & 00 & 01 & 11 & 10 \\
00 & 0 & 1 & 3 & 0 \\
01 & 4 & 0 & 5 & 1 \\
11 & 12 & 13 & 14 & 15 \\
10 & 8 & 0 & 9 & 11 \\
\end{array}
\]

\[ F = BD + A'C'D + ACD + ABC'D + A'BC \]

(b) \( F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 8, 9, 10, 11, 13) \)

\[
\begin{array}{c|cccc}
& yz \\
w x & 00 & 01 & 11 & 10 \\
00 & 0 & 1 & 3 & 2 \\
01 & 4 & 1 & 5 & 1 \\
11 & 12 & 13 & 14 & 15 \\
10 & 8 & 9 & 11 & 10 \\
\end{array}
\]

\[ F = wx' + y'z + w'y' + x'z' \]

(c) \( F(w, x, y, z) = \Pi(0, 2, 3, 8, 10) \)

\[
\begin{array}{c|cccc}
& yz \\
w x & 00 & 01 & 11 & 10 \\
00 & 0 & 1 & 3 & 2 \\
01 & 4 & 1 & 5 & 1 \\
11 & 12 & 13 & 14 & 15 \\
10 & 8 & 0 & 9 & 11 \\
\end{array}
\]

\[ F = B + C'D + AD \]