EE 451 - Exam 1
September 27, 2000

Name: ____________________________________________________________

Closed book. You may use a calculator and one page of notes. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work.

1. A continuous-time signal

\[ x_a(t) = 5\sin(600\pi t) + 2\cos(3000\pi t) - 2\sin(3400\pi t) - 3\cos(5000\pi t) \]

is sampled with a sampling frequency of 2 kHz to generate the discrete-time signal \( x(n) \).

(a) What continuous-time frequencies are present in \( x_a(t) \)?

(b) What discrete-time frequencies are present in \( x(n) \)? Be sure to normalize these frequencies to lie between \(-\pi\) and \(\pi\).

(c) What is \( x(n) \)?

(d) Is \( x(n) \) periodic? If so, what is the period of \( x(n) \)?

(e) If \( x(n) \) is passed through an ideal D/C converter (a D/A converter with an ideal reconstruction filter) what is the reconstructed signal \( y_a(t) \)?
2. Consider a system described by the difference equation

\[ y(n) = 5x(n - 1) - 4x(n - 2) + y(n - 1) + 2y(n - 2) \]

(a) What is the characteristic polynomial for this system?

(b) What is the form of the impulse response of the system? (Note: You do not need to solve for the constants such as \( C_1 \) and \( K \).)

(c) What is the transfer function \( H(z) \) of the system?

(d) Sketch the pole-zero diagram for \( H(z) \). What is the region of convergence of \( H(z) \)?

(e) Is the system stable? Explain.

(f) Find the impulse response of the system by finding the inverse \( z \)-transform of \( H(z) \)

(g) On the next page, sketch the direct-form II implementation of the system.
Direct-form II implementation of

\[ y(n) = 5x(n - 1) - 4x(n - 2) + y(n - 1) + 2y(n - 2) \]
3. Consider the pole-zero diagrams below:

Do the following without explicitly finding the inverse $z$-transforms.

(a) Sketch the approximate form of the signal $x_1(n)$.

(b) Sketch the approximate form of the signal $x_2(n)$.

(c) Sketch the approximate form of the signal $x_3(n)$. 
4. When the input to a linear time-invariant system is

\[ x(n) = 5u(n), \]

the output is

\[ y(n) = \left[ 2 \left( \frac{1}{2} \right)^n + 3 \left( -\frac{3}{4} \right)^n \right] u(n). \]

(a) Find the system function \( H(z) \) of the system.

(b) Plot the poles and zeros of \( H(z) \), and indicate the region of convergence.

(c) Write the difference equation which characterizes the system.

(d) Is the system stable? Is it causal? Explain.