Consider the second order differential equation \( \ddot{y} + 2 \zeta \omega_n \dot{y} + \omega_n^2 y = K \omega_n^2 u \) with \( y \) the output, \( u \) the input, \( \omega_n \) the undamped natural frequency, \( \zeta \) the damping ratio, and \( K \) the DC gain.

1. Represent the system in both state-space form and as a transfer function.
2. Show that the poles/eigenvalues are \( s_{1,2} = -\sigma \pm j \omega_d \) where \( \sigma = \zeta \omega_n \) is the damping factor and \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \) is the damped natural frequency.
3. Solve for step response assuming an underdamped system, i.e., that \( s_{1,2} \) are complex.
4. Show that the steady-state value of \( y \) is \( K \).
5. Show that the time to the maximum (peak) is \( T_p = \frac{\pi}{\omega_d} \) and that the overshoot is \( M_p = K e^{-\zeta \pi \sqrt{1 - \zeta^2}} \).
6. Show that the 1% settling time is \( T_s \approx \frac{4.6}{\sigma} \). What would the 5% settling time be in terms of \( \sigma \)?
7. Rise time \( T_r \) is a little more difficult to relate to poles/eigenvalues. Search around and see what you can find for a relationship (most often approximated as an average from range of responses). \( T_r \approx \frac{1.8}{\omega_n} \) is one I've seen used.
8. Pick values, say \( K = 2, \omega_n = 4, \zeta = 0.4 \), such that response is underdamped and plot response (either numerically solve DE or directly plot \( y(t) \)). Confirm metrics by matching values on plot to calculated/expected values.